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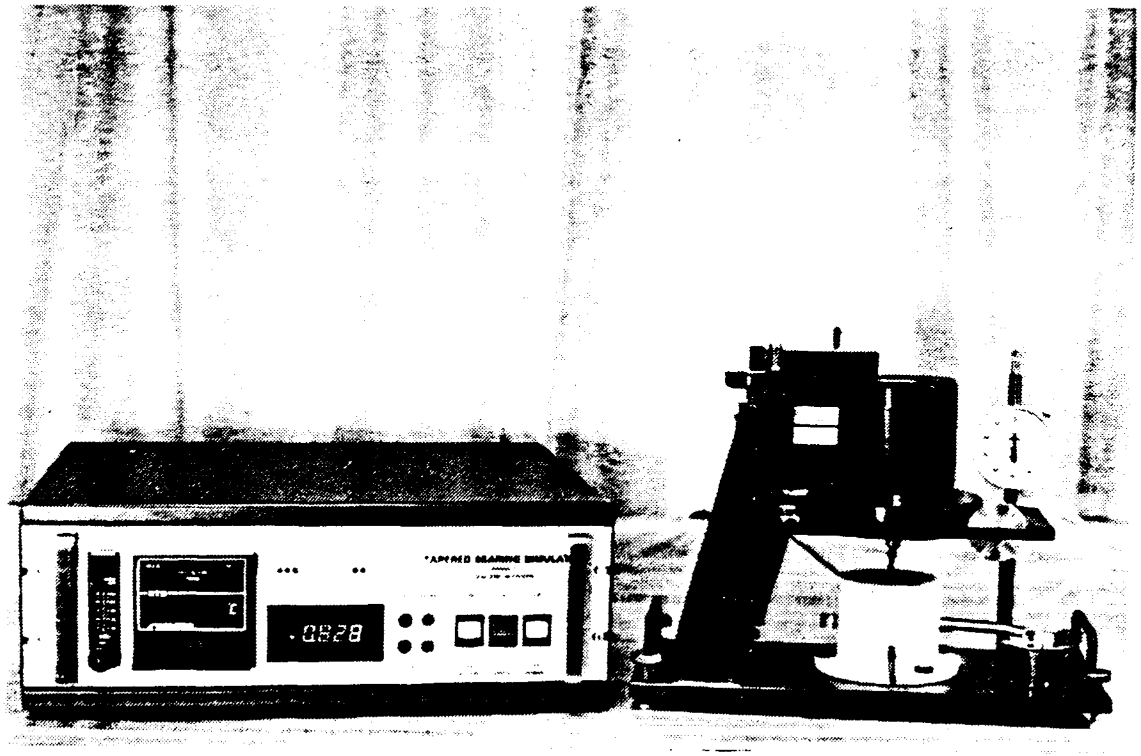

Fig. 1 - The Tapered Bearing Simulator, Model 400.

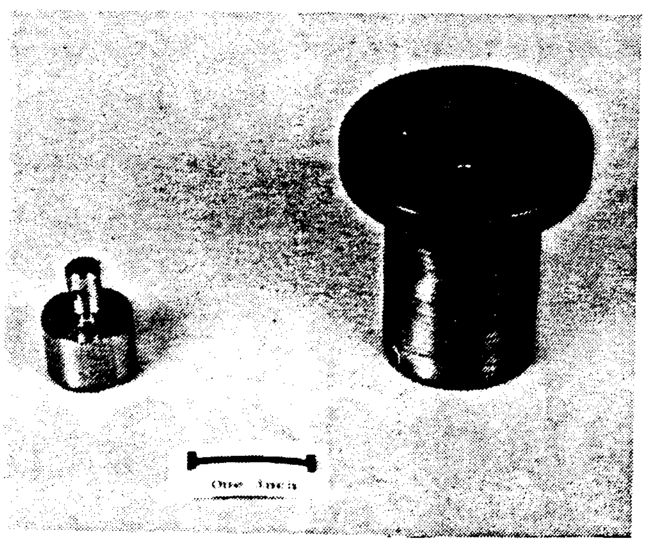

Fig. 2a - Model 400 rotor and stator

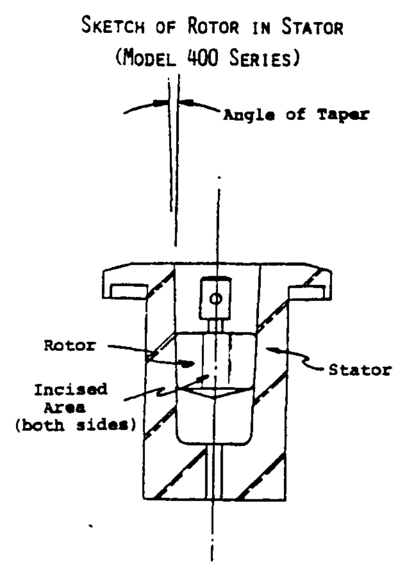

Fig. 2b - 'Cutaway' sketch of rotor in stator

ABSTRACT

The high shear, high temperature viscometry of non-Newtonian oils requires absolute viscometry or reference fluids from such viscometry to use with relative instruments. To be sufficiently precise, such absolute viscometry should be established at the temperature and shear rate of concern. The authors show how the Tapered Bearing Simulator meets the above criteria of performance. The instrument is then applied to the international reference oils established by the ASTM and CEC and the results seem to indicate that previously generated viscometry values may be in considerable error regarding the apparent shear rate applied.

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THE TAPERED BEARING SIMULATOR, shown in Figure 1, is a rotational viscometer developed for high shear, high temperature analyses. Its development has been detailed in another paper (1). The name was derived both from the slightly tapered configuration of the rotor/stator pair shown in Figure 2 and the similarity of the basic geometry of the pair to that of an automotive journal and bearing whose operational shear rate the viscometer was meant to simulate. The application of the viscometer is not, of course, limited to automotive lubricants although this has been one of the prime areas of interest in its application.

Surprisingly, the slightly tapered geometry has been infrequently chosen (1,2,3,4) for a variable shear rate instrument. Fine control of the rotor/stator gap is inherent in the slight taper where in the present instrument, an axial adjustment of 160 units is required to obtain a radial change of 1 unit. Such control is very desirable when the gap is only the 3 or 4 microns required for very high shear viscometry.

Originally, the Tapered Bearing Simulator was intended for use as a 'relative' rotational viscometer (one requiring calibration with reference oils of known viscosities). As such, the instrument obeys Newtonian principles very closely. For example, Figure 3 shows a typical calibration using five reference oils. The

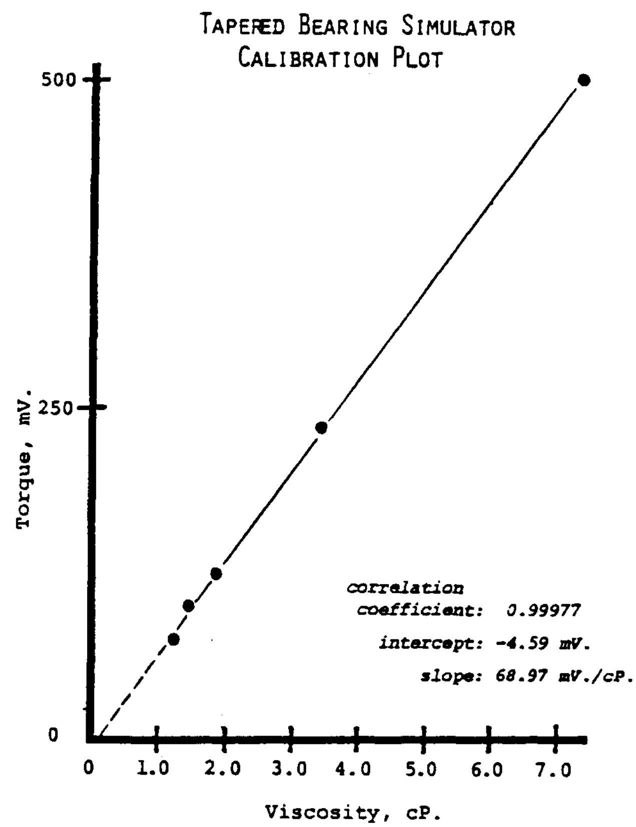

Fig. 3 - Linearity of calibration curve for Tapered Bearing Simulator

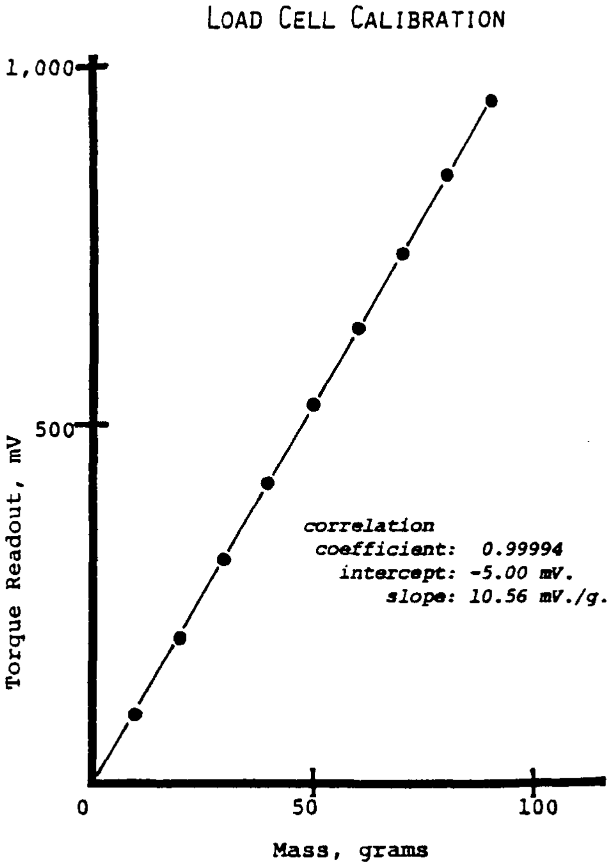

Fig. 4 - Linearity of Load Cell calibration

degree of linearity of such data is a measure of the degree to which the instrument being evaluated can perform as a true viscometer. When these data are linearly regressed, a Correlation Coefficient of 0.9998 is found with a sensitivity of 69 mV/cP. A sufficiently high level of performance as a viscometer.

Part of the reason for the instrument's sensitivity and linear performance is to be found in the load cell which responds to the shearing forces generated in the fluid by the driven rotor. The load cell gives a very linear response as shown in Figure 4. Linearity is highly important to precision, and such linearity is evident from the Correlation Coefficient of 0.99994.

Since the Tapered Bearing Simulator showed such encouraging levels of linearity and sensitivity, it seemed reasonable to investigate whether the instrument could perform as an absolute rotational viscometer. The question was more than academic since many oils to which the instrument would be applied would be non-Newtonian. Relative instruments thus require non-Newtonian reference oils in order to determine the shear rate at which the instrument is operating and such oils are not available as standards. This situation was, and is, a problem in using any instrument as a relative viscometer.

THE TAPERED BEARING SIMULATOR AS A RELATIVE VISCOMETER AT HIGH SHEAR - Two approaches were used to establish the shear rate for the Tapered Bearing Simulator when studying non-Newtonian oils.

The first was to chose a non-Newtonian fluid which had been previously analysed at a suitably high shear rate in an absolute concentric cylinder viscometer. Then the viscometric value of the fluid chosen as a standard was reproduced in the TBS/Viscometer after the instrument had been calibrated with Newtonian oils as in Figure 3. The fluid chosen was the CEC Reference Oil RL64/2 and the absolute instrumental results were from the values reported by duParquet using the instrument developed at the TOTAL Centre de Recherches (5). The value chosen as representative of this non-Newtonian oil at about one million reciprocal seconds was 4.2cP.The torque level required to accomplish this response was noted and used in subsequent analyses to establish the rotor/stator gap.

The second approach was semi-absolute in that the authors used the calculated torque required for a Newtonian fluid of known viscosity if the rotor/stator gap were that required for one million reciprocal seconds shear rate. This approach required assigning a value to the shearing area on the rotor. It was found that by assuming the incised areas on the rotor were not functioning as part of the shearing area, values were obtained on the various ASTM and CEC reference oils which were similar to those being reported by other groups.

A DISCUSSION OF ABSOLUTE VISCOMETRY may be of value to the reader at this point.

First, to review what has already been said, most viscometers are considered relative instruments since they must be calibrated by reference fluids whose viscosities have already been established using absolute viscometers. As previously noted, the latter are viscometers from whose configuration the viscous response of an unknown fluid may be calculated according to equations appropriate to the particular geometry of the instrument. It is understandably desirable to be able to avoid the need to calibrate a viscometer particularly when the temperature, shear or fluid composition are such that there are few non-Newtonian fluids available or known with which to perform such calibration. More important, the absolute instrument permits the direct determination of the shear stress and shear rate — values which are necessary to determine or establish other conditions of analysis for non-Newtonian fluids.

Practical Considerations in Absolute High Shear Viscometry -

The primary problem in absolute viscometry, particularly at high shear, is to really know the shear rate and shear stress at which the viscometer is operating. A number of viscometers stated to be absolute are so only at or near the temperatures at which their dimensions have been measured — commonly room temperature and low shear rates (to avoid the generation of viscous heat which would be certain to, in some degree, change the critical dimensions of the instrument by bringing on the effects of thermal expansion). Unless the coefficient of expansion of the materials of construction are inordinately low, such dimensional changes at the small dimensions associated with high shear rate measurements are very difficult to sustain and still permit precise use of a given instrument as an absolute viscometer, particularly at temperatures much removed from room temperature. Thus, the consequences are that absolute viscometers of known configuration at room temperature are often used as relative instruments at other temperatures and high shear rates. Even if the expansion coefficients of the materials of construction are known and applied, the geometrical analysis of the resultant configuration can leave considerable uncertainty. Furthermore, such calculation must assume knowledge of the precise temperature profile of the materials through which shear is imparted to the fluid — a further difficulty in generating the required geometrical information.

The result of this situation is that at high temperatures and shear, the true shear rate in almost all so-called absolute viscometers is known but approximately. Such approximations may be adequate for some applications. On the other hand, such approximations may be inadequate for the precise determination of the viscosity of non-Newtonian fluids when dependence on the high shear viscometric properties of the fluid is critical, as in engine oils where hydrodynamic lubrication is highly viscosity dependent.

THE GEOMETRY OF SHEAR - Over and above the foregoing considerations are the fundamental differences among the configurations of viscometers and the effect of these configurations on the information obtained.

Essentially, there are two fundamental approaches to the measurement of viscosity:

1) measuring the flow rate of the fluid under some applied force, or

2) measuring the resistance offered by the fluid to some surface moved in contact with the fluid.

The first approach leads to capillary and orifice instruments, the second to rotational instruments, one of which, of course, is the concentric cylinder configuration important to the considerations in this paper.

When used to measure non-Newtonian oils at high shear and temperature, both instrument geometries can and do seem to give reasonably equivalent results. This may be, in fact, somewhat misleading, depending on the way in which the shear rate in capillary viscometers is calculated.

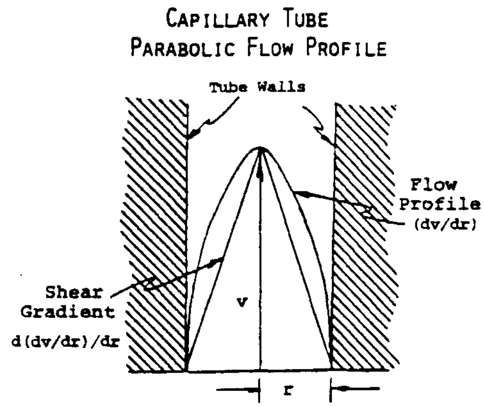

Fig. 5 - Flow parameters for capillary tube

Capillary instruments develop a parabolic shear profile (shown in Figure 5) and a linear shear gradient (i.e. change in shear rate) which ranges from zero at the axis of the capillary to some maximum value at the capillary wall dictated by the driving pressure. The viscosity values produced by these instruments are often reported at the maxium shear rate at the wall layer. Kroepelin (6) showed that the averge shear rate in the capillary viscometer was 2/3rds of the maximum (for a Newtonian fluid). In contrast, for a concentric cylinder viscometer of very narrow gap, the profile is linear (see Figure 6) and the gradient is zero, that is, the shear rate does not change across the gap.

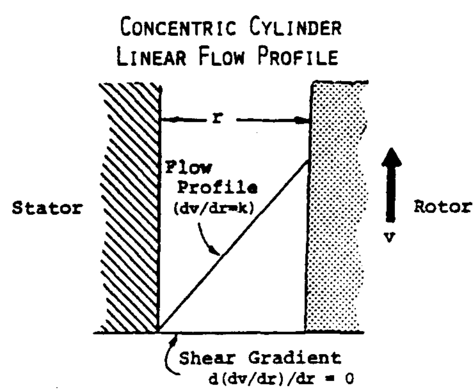

Fig. 6 - Flow parameters for concentric cylinders

From the foregoing it is evident that the viscosity of a non-Newtonian fluid would be expected to appear higher when measured in a capillary viscometer than in a concentric cylinder viscometer compared at the same shear rate if the shear rate at the wall of the capillary is used for the comparison. The fact that absolute instruments of the two kinds seem to agree so well may suggest that perhaps the values reported from concentric cylinder viscometers at high shear rates and temperatures may be lower than expected as a consequence of the previously mentioned thermal effects. Or it may be that since Kroepelin's work only applies to parabolic flow profiles, non-Newtonian flow conformations may place the average shear rate nearer the capillary walls.

THE TAPERED BEARING SIMULATOR AS AN ABSOLUTE VISCOMETER - With the background given, it is reasonably apparent why the authors made the effort to determine whether the Tapered Bearing Simulator could be used as an absolute viscometer. It was evident that the ability to change the gap between the rotor and stator during operation should permit a uniquethe application of Newtonian's laws of fluid motion to determine the absolute gap and thus whether or not the incised areas were operative. The relationship between the gap and the reciprocal of torque had been noted many years previously in an interesting study by Needs (3) using the Kingsbury Tapered Plug Viscometer.

CALCULATIONS CONCERNING THE TAPERED BEARING SIMULATOR - ARGUMENT - The Tapered Bearing Simulator has such a slight taper to the rotor and stator that if the taper were the only factor, for all comparitive purposes and calculations, the instrument could be considered a concentric cylinder viscometer. However, another factor is present: the rotor of the instrument has opposing incised areas on the virtually cylindrical surface as shown in Figure 2b. Does the presence of these incised areas reduce the shearing area of the rotor? The immediate answer would seem to be 'yes' since it is evident that rotor surfaces underlying the incised areas are generally at a relatively greater distance from the opposing walls of the stator. If these incised surfaces themselves were operative, the shearing forces for these areas would, of course, have much less contribution to make to the total shearing force than the equivalent area running in close proximity to the stator wall. A more considered answer to the foregoing question, however, might be 'perhaps not'. Considering that the fluid within the incised area may be trapped in a circulating pool within the incised volume, what is to prevent the transfer of shearing energy generated by the rotor through this trapped fluid to the laminar flow regin nearer the stator wall? In this case, the trapped fluid volume would function as a section of the rotor's surface of closest approach. This is, the rotor would operate as though the incised areas were not present -- the rotor would be 'solid'.

While it would be very difficult to resolve the question of rotor shear area by mathematical analysis, it could be resolved very quickly by experimentally determining the forces exerted by the rotor on a fluid of known viscosity at a known shear rate. To accomplish this would essentially require establishing a known shear rate and then measuring the shearing force for the area of the rotor. That is to say, use the Tapered Bearing Simulator as an absolute viscometer. Alternatively, the rotor/stator gap could be set on the basis of the calculated shearing forces with and without the participation of the incised areas and this gap compared to the gap determined by the reciprocal torque relationship. The latter approach was the one chosen.

Such a study would, understandably, require good precision free of other interferences. The study had the potential of teaching a considerable amount about the nature of high shear viscometry and the effect of materials and geometry of the rotor/stator pair.

Calculation of the Shearing Force - The shearing force can be calculated directly from the dimensions of the rotor, the length of the torque arm pressing against the anvil of the load cell, (Figure 7) the speed of the rotor, the known viscosity of the fluid, and the assumed film thickness. As usual, the initial equation is Newton's (7)

$$[1] \qquad \eta = \tau G$$

Equation 1 can be restated as

$$[2] \qquad F/A = \eta\, v/h$$

for the case in which the rotor/stator gap is so small that the slight curvature (in comparison to the film thickness) may be considered negligible — calculated to be less than 0.07% error (3). Using the critical dimensions of the Tapered Bearing Simulator given with the Nomenclature at the end of the paper, and assuming a 'solid' rotor (i.e. one without incised areas), the shear stress at the rotor

Rotor Radius and Turntable Torque Arm

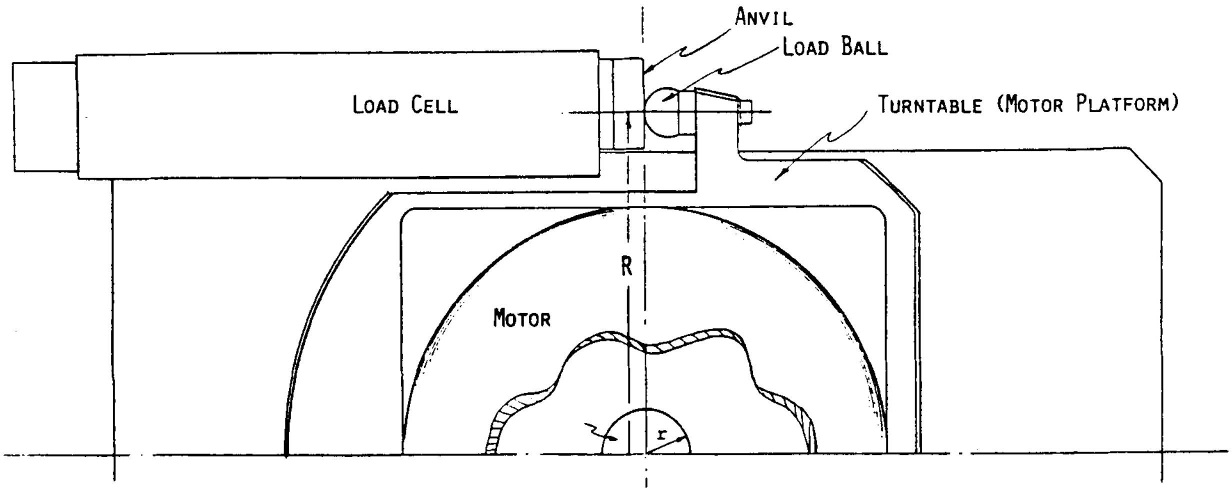

Fig. 7 - Diagram of rotor radius and length of torque arm

surface is

$$[3] \quad F/A = \eta (T/r)/2\pi rL$$
$$= \eta T/2\pi r^2 L$$

in which

$$[4] \quad T = Fr.$$

If R is the turntable torque arm, shown in Figure 7,

$$[5] \quad T = F*R.$$

At one million reciprocal seconds shear rate for a fluid having a viscosity of 10cP, the shearing force at the anvil can be shown to be

$$[6] \quad F'' = (r/R)A\eta v/h$$
$$= 96{,}500 \text{ dynes.}$$

If the incised areas were not operative, the shearing force would be

$$[7] \quad F' = (r/R)A'\eta v/h$$
$$= 74{,}800 \text{ dynes}$$

about 22% less shearing force — a difference which should be able to be detected by a reasonably precise viscometer operating at a known shear rate.

Establishing the Shear Rate

In addition to permitting the variation of shear rates by varying the rotor/stator gap, the taper also has the very desirable property of permitting the precise determination of the gap at the operating temperature and shear rate. Needs (3) published a complete description of the technique using the Kingsbury Tapered Plug Viscometer. Essentially, the technique utilizes the relationship explicit in Equation 2. If viscosity, speed, and rotor area are constant, then

$$[8] \quad F = k/h$$

in which k is a proportionality constant related to the sensitivity of the instrument. Equation 8 is a linear relationship. Thus, as sketched in Figure 8, a plot or a regression analysis of the reciprocal of the force, 1/F, versus the reading of the height-indicating micrometer should give a straight line intersecting the height axis at the theoretical point where h becomes zero. Such a linear relationship would only be shown if the viscometer strictly obeyed Newtonian laws and if there were no other interfering factors such as film temperature variation with film thickness.

It should also be mentioned that the theoretical contact point of the surfaces most likely refers to the surface represented by the immobile fluid nearest to the metal surfaces as shown in Figure 9. Actually, of course, metal-to-metal contact should occur before this theoretical point is reached as the asperities of the rotor and stator protruding through the immobile fluid come into occlusion.

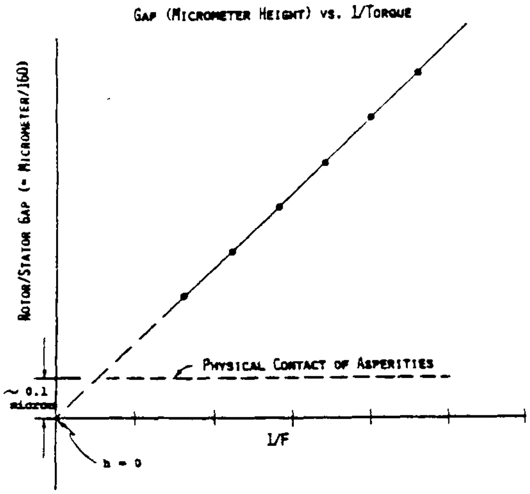

Fig. 8 - Illustration of how the theoretical contact of rotor and stator can be found

EXPERIMENTAL DETERMINATION OF THE ROTOR/STATOR GAP were conducted at 150 C. After establishing the rotor and stator at this temperature, the rotor was first carefully 'grounded' (that is, brought into light contact with the stator while rotating it by hand) to determine the upper limits of the possible contact zone. The rotor was then raised about 1.5 mm and the sequence of data collecting initiated. The results are shown in Figure 10 and it contains several pieces of interesting information.

First of all, the plotted data essentially form a straight line with a slight departure at higher gaps. Assuming that the slight departure is the onset of thermal effects (which would be expected to be more evident at greater film thickness), the final 10 points - from 1 mm and less - give a correlation coefficient of 0.9998. This is respectable evidence that

1) thermal effects generated by shearing the fluid are minimal, particularly at the smaller gaps.

2) the h=0 intercept should be able to be determined fairly precisely.

3) the apparent contact point of the rotor and stator as determined by hand rotation is 2.6 microns above the theoretical intercept and suggests that, without the centering forces of high speed rotation, the apparent contact point reflects some degree of cocking of the rotor in the stator in addition to asperity contacts.

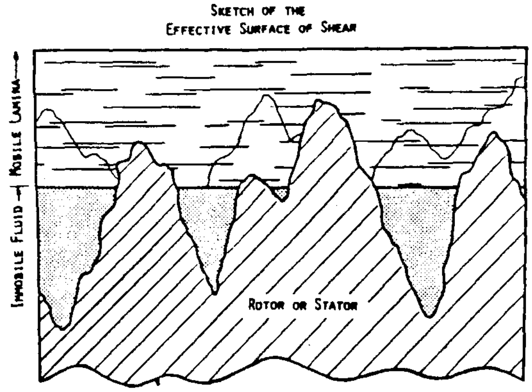

Fig. 9 - Sketch of immobile and mobile fluid layers with surface asperities

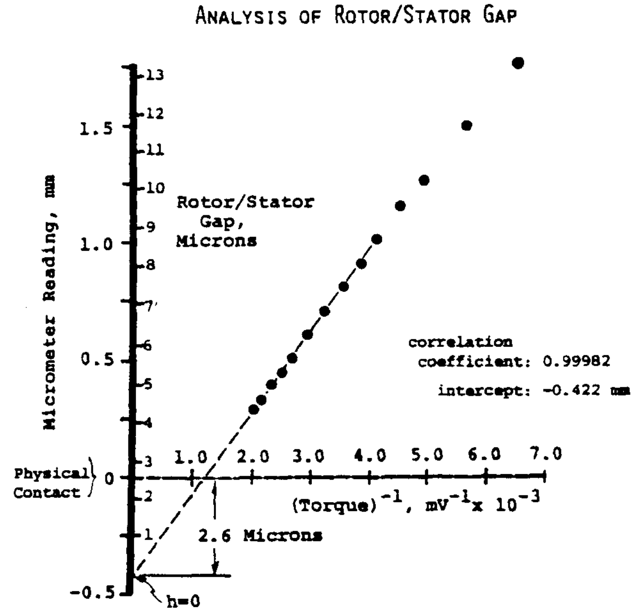

Fig. 10 - Actual Determination of theoretical contact and rotor/stator gap

From the results, it is clear that the taper of the rotor/stator pair can be used to set the gap between them and, thus, set or determine the shear rate while the instrument is operating at high temperature and shear rate.

To determine the value of the intercept more exactly, fluids of different viscosities or different rotational speeds can be used in order to 'triangulate' the intercept value. Needs (3) demonstrated this as shown in Figure 11 and the authors have independently corroborated his work. Essentially, the method uses the intercepts found with two or more speeds or fluids and averages the values so determined. In the authors' initial work, usually the values agreed within 0.03mm for a precision in establishing the gap dimension of +/-0.10 micron. Later work and the studies by Needs (3) suggest this can be improved.

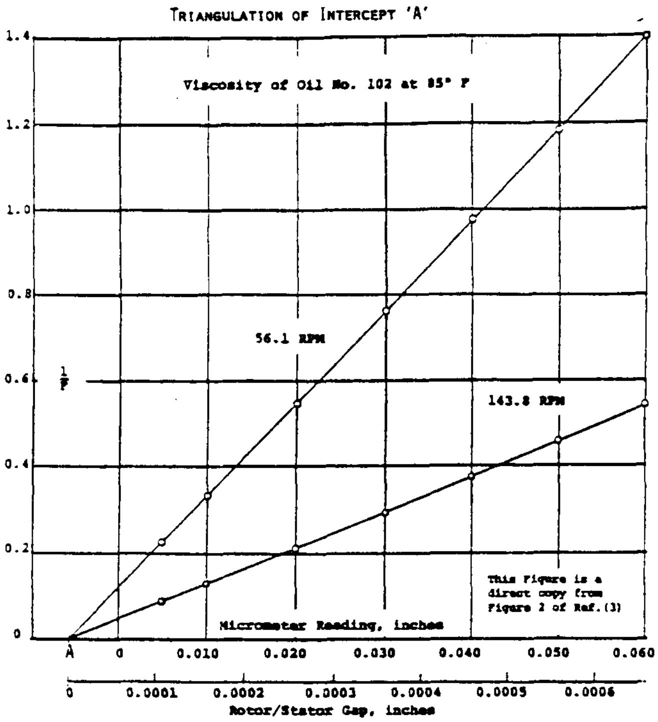

Fig. 11 - 'Triangulation' of contact point.

EXPERIMENTAL DETERMINATION OF THE EFFECTIVE ROTOR SHEARING AREA - Determination of Rotor/Stator Gap Using the Rotor Shearing Area and Required Force - The authors used the ASTM Newtonian Reference Fluid R-600 at 150 C and 3600 rpm and set the rotor to the position at which the viscosity of this fluid would produce the torque required. The viscosity of R-600 is 7.26cP at 150 C. Thus, if the incised areas are non-operative, the shearing force required to produce one million reciprocal seconds shear rate, according to Equation 7, is.

$$7.26cP/10cP = x/74{,}800 \text{ dynes}$$

$$\text{and } x = 54{,}300 \text{ dynes}$$

In this case, using a rearrangement of Equation 7, the gap will be

$$[9] \quad h'' = (r/R)A' \quad V/F''$$

$$= 3.48 \text{ microns}$$

If the incised areas are, on the other hand, operative, then Equation 9 becomes

$$[10] \quad h' = (r/R)A \quad V/F''$$

$$= 4.47 \text{ microns}$$

Returning to Figure 10, it will be remembered that the rotor was set at the shearing force necessary to generate one million reciprocal seconds if the incised areas were not operative. The theoretical intercept was found at -422 microns below the apparent intercept and the position of the rotor at the proper torque level is 282 microns above the apparent physical intercept. This gives a total vertical height of 704 microns above the h=0 intercept and, thus, a gap of

$$704/160 = 4.40 \text{ microns}$$

a value very close to that calculated in Equation 10 for the rotor when the incised areas were operative. Briefly stated, the rotor operates as though it had a solid shearing surface and the trapped fluid in the incised areas apparently do transfer force equivalent to a solid wall.

At 4.40 microns the actual shear rate in the rotor/stator gap is 788,000 reciprocal seconds.

NEW HIGH SHEAR RATE STUDIES OF THE CEC AND ASTM REFERENCE FLUIDS at the shear rate of one million reciprocal seconds using the Tapered Bearing Simulator as an absolute viscometer are presented in Table 1 in contrast to the values found previously at lower shear rates. It is apparent that, with the exception of the Newtonian fluids, the higher shear rate representing a presumably true value of one million reciprocal seconds produce significant changes in the values of the non-Newtonian fluids.

TABLE 1.

Viscosities of ASTM and CEC Reference Oils at Two Shear Rates

REFERENCE OILS	VISCOSITY, cP. 0.78×10^6 sec^{-1}	1.0×10^6 sec^{-1}	Δ %
HTR 30	4.41	4.21	4.5%
HTR 31	3.44	3.25	5.5%
HTR 32	3.86	3.50	9.3%
HTR 33	3.70	3.46	6.5%
HTR 34 (Newtonian)	3.51	3.47	1.1%
HTR 35	3.43	3.27	4.7%
HTR 36 (Newtonian)	2.34	2.39	-2.1%
HTR 37	2.75	2.56	6.9%
HTR 39	3.01	2.74	9.0%
RL 59	4.20	3.83	8.8%
RL 65	4.83	4.49	7.0%
RL 88	3.36	3.16	6.0%
RL 90	4.14	3.84	7.2%
RL 91 (Newtonian)	1.96	2.02	-3.1%

From a recent paper (8) of the CEC L-23 group, a limited set of data from a capillary instrument is compared to the Tapered Bearing Simulator results in Table 2.

TABLE 2.

Comparison of Capillary and Tapered Bearing Simulator Viscometers

REFERENCE OILS	VISCOSITY, cP.		
	CAPILLARY	TBS	
	1.0×10^6 sec^{-1}	0.78×10^6 sec^{-1}	1.0×10^6 sec^{-1}
RL 88	3.25	3.36	3.16
RL 90	3.94	4.14	3.84
RL 91 (Newtonian)	2.06	1.96	2.02

DISCUSSION

On the basis of these studies it seems as though the tapered geometry, sensitivity, and temperature control of the Tapered Bearing Simulator combine to form a somewhat unique absolute viscometer. The uniqueness is associated with the fact that the shear rate of the instrument is determined while in operation whereas other absolute instruments are based on their initial physical measurements and, where necessary, associated correction terms. The actual measurement of the rotor/stator gap by Newtonian laws of flow has much to recommend it, particularly as it eliminates the uncertainties connected with the thermal effects at high shear and temperatures. It is not unreasonable to conjecture that, although more difficult to fabricate, p-erhaps many of the concentric cylinder absolute viscometers would benefit from alteration to a slight taper and axial motion of the rotor or stator.

Regarding the significance of of the authors' analysis of the CEC and ASTM non-Newtonian reference fluids, the Tapered Bearing Simulator has brought the previously accepted values into serious doubt, at least at the nominal level of one million reciprocal seconds. From the data presented in this paper, it would seem more likely that the past data have been collected at about three-quarters of a million reciprocal seconds. The difference in the values obtained on these fluids taken at these two shear rates are not minor. Some of the oils showed almost 10% reduction in their measured viscosity as a consequence of the change in shear rate. Since the ASTM/CEC round-robin on evaluating the precision of the Tapered Bearing Simulator and other viscometers is just beginning, if the authors' data is considered persuasive, it is reasonable to consider either the use of the actual shear of one million reciprocal seconds or the acceptance of the lower shear rate for measurement.

It was also of interest to note that up to a rotor/stator gap of more than 10 microns, no evident shearing heat effects are observed. This has been one of the nagging questions regarding concentric cylinder viscometry at high shear rates. The present experience and further studies may lay this concern to rest, at least as far as rotor/stator gaps less than about 10-12 microns are concerned. However, gaps of these dimensions may be somewhat difficult to obtain in any but tapered geometries.

Although the data are meager, some comment should be made concerning Table 2. The agreement between the capillary viscometer and the Tapered Bearing Simulator was reasonably close although there is more difference between the non-Newtonian oils, 88 and 90, than the Newtonian oil, 91. Moreover, the differences were consistent. On the basis of this very limited comparison, it would seem as though the average shear rate in the capillary is less than the one million reciprocal seconds noted in the publication.

ACKNOWLEDGMENTS

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NOMENCLATURE

A = Area of solid rotor (7.34 cm^2)
A' = Area of rotor without incised areas (5.69 cm^2)
F = Force at rotor surface
F* = Force at end of torque arm
F' = Force associated with use of A' in calculations
F'' = Force associated with use of A in calculations
G = Shear Rate
h = rotor/stator gap
h' = gap associated with use of A' in calculations
h'' = gap associated with use of A in calculations
k = proportionality constant
L = rotor length (1.27 cm)
R = torque arm (7.00 cm)
r = rotor radius (0.92 cm)
T = Torque
V = linear velocity of rotor surface (347 cm/sec)

π = pi
η = dynamic viscosity
τ = shear stress